Deciding the First-Order Theory of an Algebra of Feature Trees with Updates

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The CoLiS Project

- ANR project with IRIF, Inria Saclay, Inria Lille.

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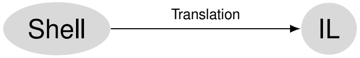

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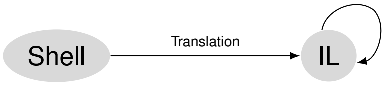

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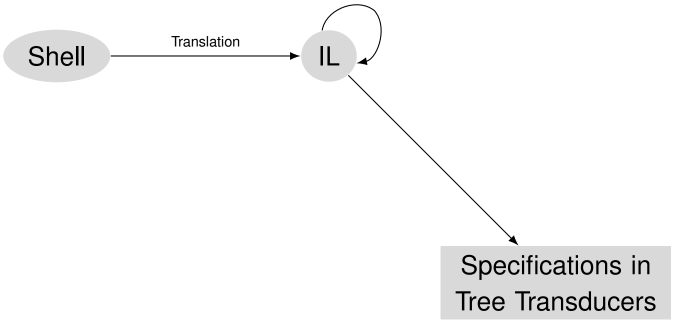

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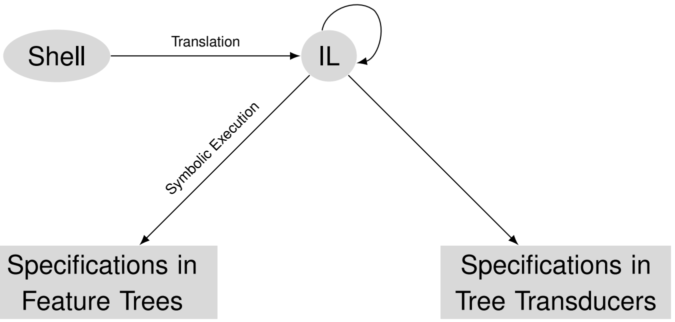

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Symbolic Execution
Specifications in Feature Trees
Specifications in Tree Transducers

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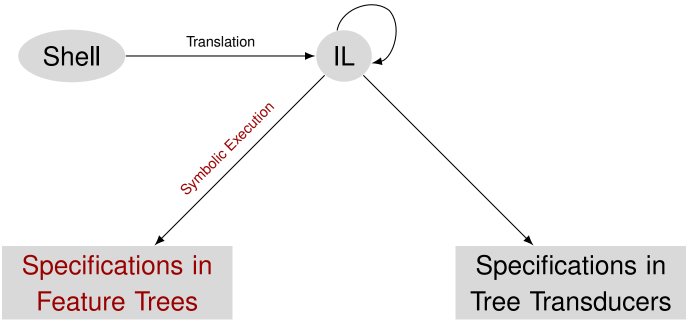
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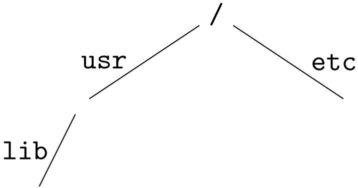
  - Sequence of scripts that do nothing:
    $\forall in, out \cdot (\exists r \cdot (\texttt{spec}_s(in, r) \wedge \texttt{spec}_t(r, out)) \leftrightarrow out \doteq in)$

Feature Trees and Update

Unix Filesystem



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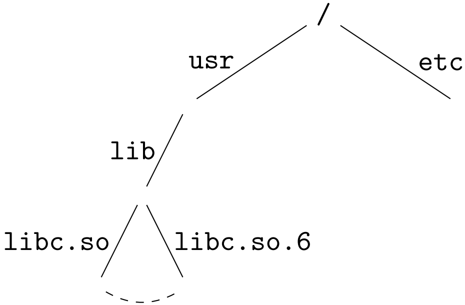

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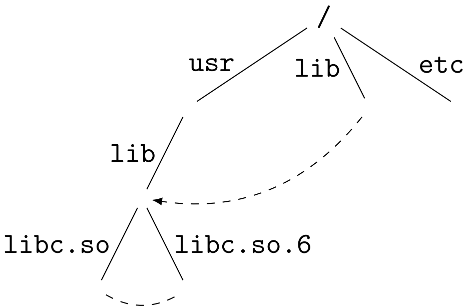

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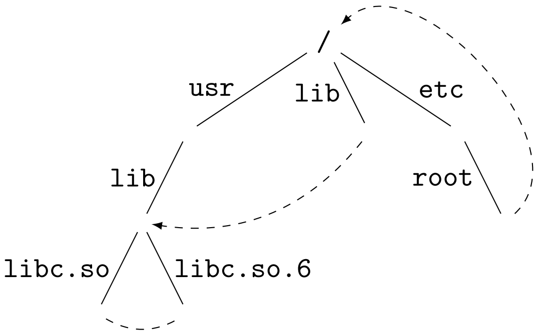

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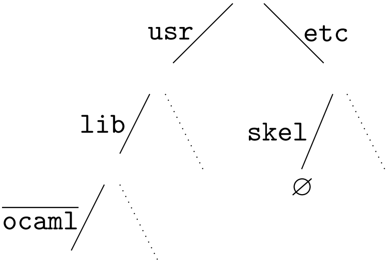

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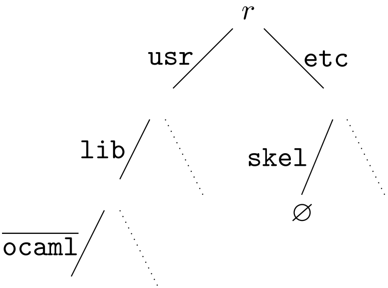

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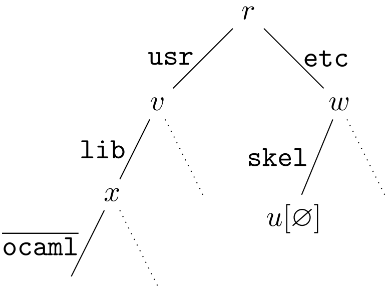

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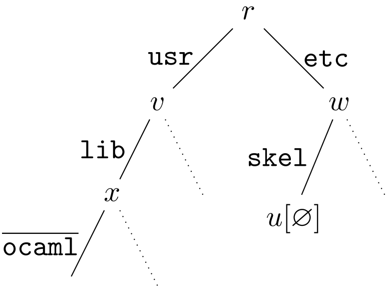

$$c(r) \ = \ \exists u, v, x, w \cdot \begin{cases} r[\texttt{usr}]v \wedge v[\texttt{lib}]x \\ \wedge\, r[\texttt{etc}]w \wedge w[\texttt{skel}]u \end{cases}$$

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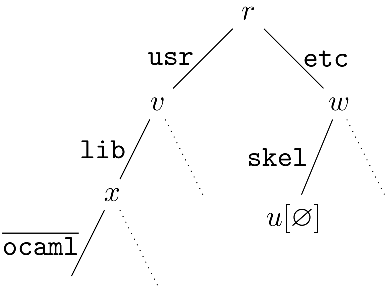

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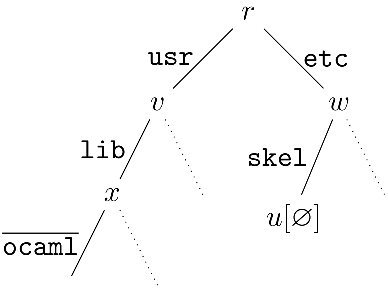

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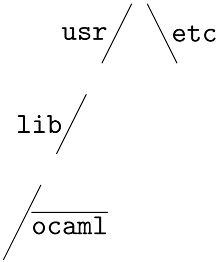

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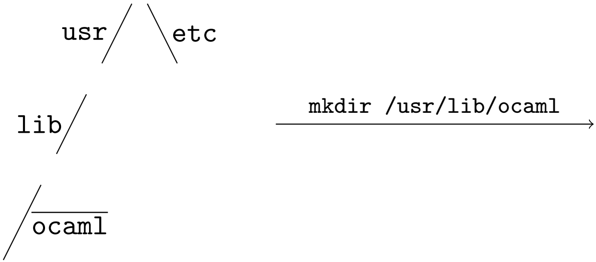

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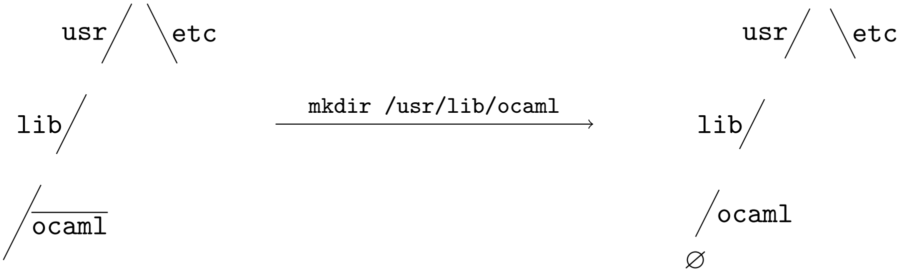

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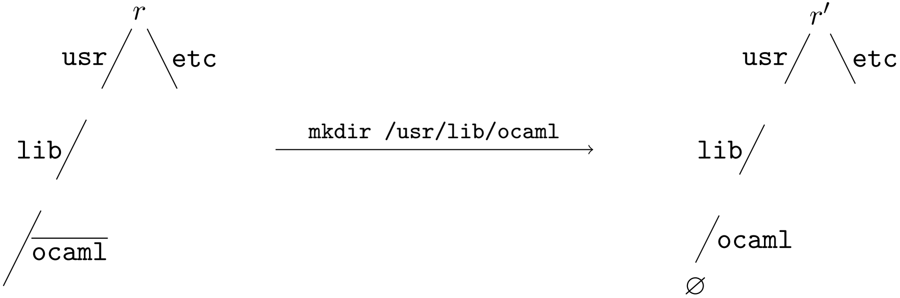

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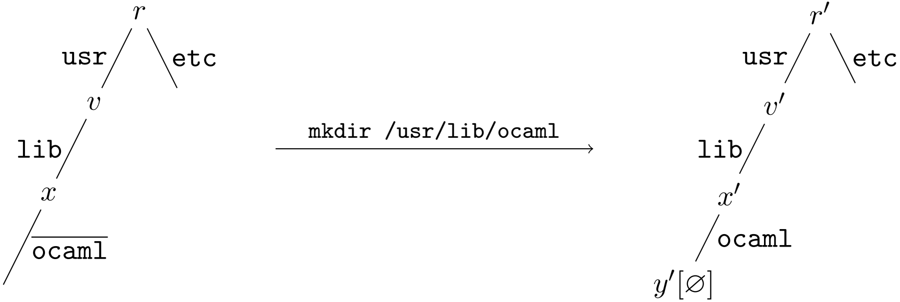

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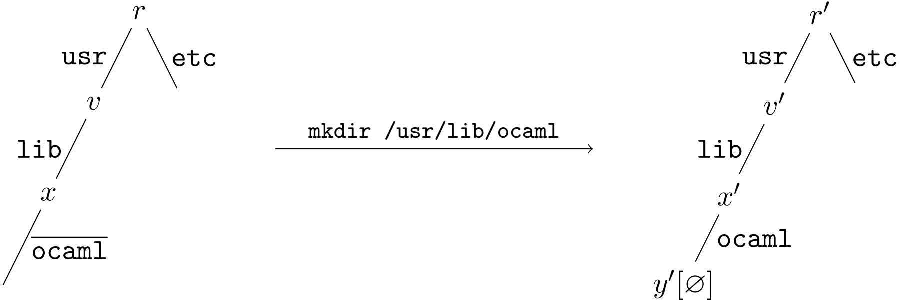

$$c'(r, r') \;=\; \exists v, v', x, x', y' \cdot \begin{cases} r' \text{ is } r \text{ with } \texttt{usr} \to v' \\ \wedge\ v' \text{ is } v \text{ with } \texttt{lib} \to x' \\ \wedge\ x' \text{ is } x \text{ with } \texttt{ocaml} \to y' \\ \wedge\ y'[\varnothing] \end{cases}$$

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Model and Examples

$$\mathcal{FT} = \mathcal{F} \rightsquigarrow \mathcal{FT}$$

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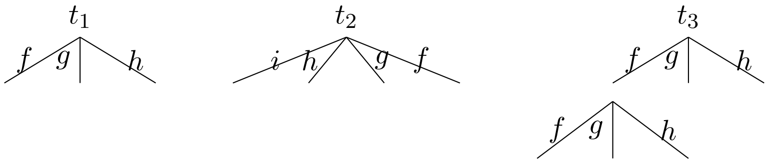

Constraints and their Interpretation

Equality	$x \doteq y$
Feature	$x[f]y$
Absence	$x[f]\uparrow$
Fence	$x[F]$
Similarity	$x \sim_F y$

- x, y variables.
- $f \in \mathcal{F}$, $F \subset \mathcal{F}$ finite.

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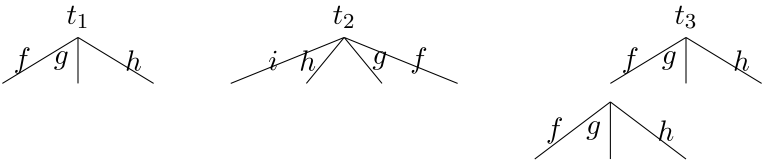
Fence	$\mathcal{FT}, \rho$	$\models$	$x[F]$	iff	$\mathtt{dom}(\rho(x)) \subseteq F$
Similarity	$\mathcal{FT}, \rho$	$\models$	$x \sim_F y$	iff	$\rho(x) \upharpoonright \overline{F} \;=\; \rho(y) \upharpoonright \overline{F}$

- x, y variables.
- $f \in \mathcal{F}$, $F \subset \mathcal{F}$ finite.
- ρ a valuation from variables to $\mathcal{FT}$.

Examples (Again)



The following constraints are satisfied in $\mathcal{FT}, [x \to t_1, y \to t_2, z \to t_3]$:

$$z[f]x, \quad x[i]\uparrow, \quad x[\{f,g,h,i\}], \quad x \sim_{\{i\}} y, \quad x \sim_{\{h,i\}} y$$

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- Normal form: incremental.
- The rules come from properties of the constructions.

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Clash Rules

C-FEAT-ABS $x[f]y \wedge x[f]\uparrow$

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Simplification Rules

S-FEATS
$$\exists X, z \cdot (x[f]y \wedge x[f]z \wedge c)$$
$$\Rightarrow \exists X \cdot (x[f]y \wedge c\{z \mapsto y\})$$

Rules with the Similarity Constraint

Propagation Rules

P-FEAT
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$$\Rightarrow x \sim_F y \wedge x[f]z \wedge y[f]z \wedge c \qquad (f \notin F)$$

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P-SIM
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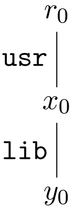
- We can “garbage collect” the normal forms to make them smaller.

Garbage Collection



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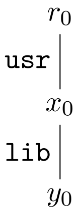

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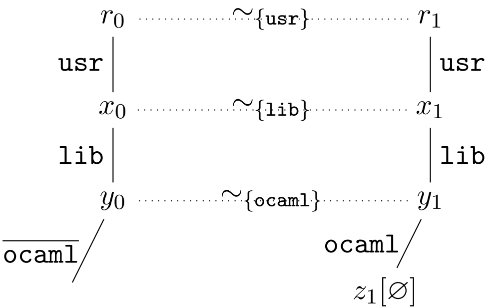

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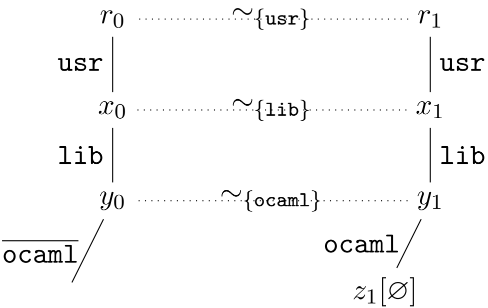

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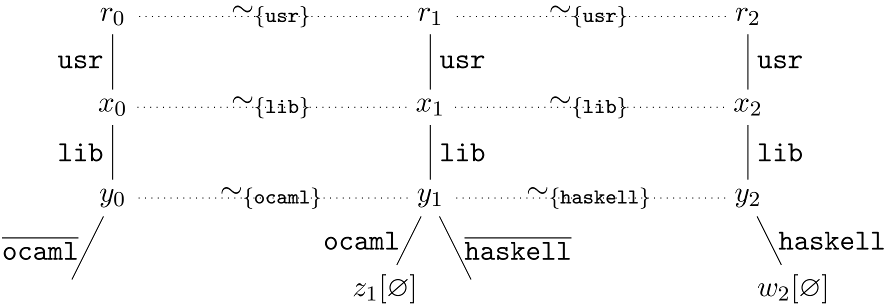

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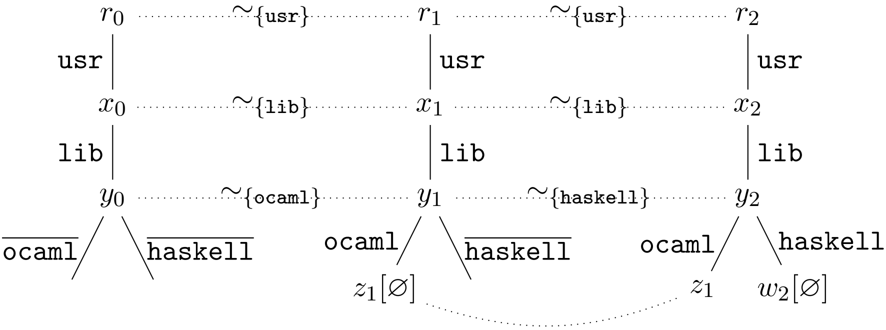

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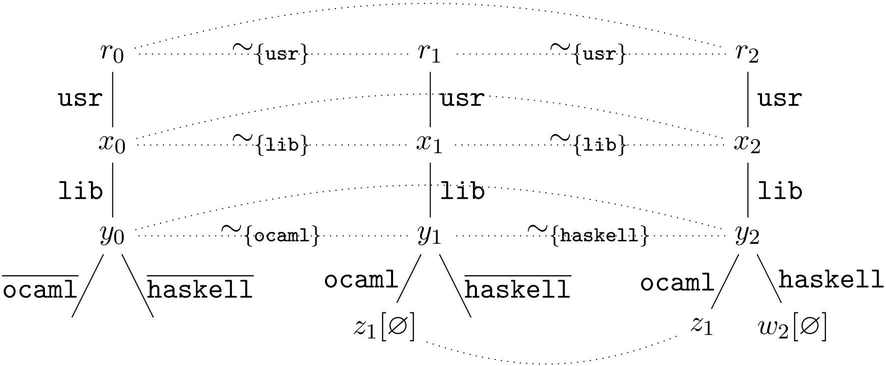

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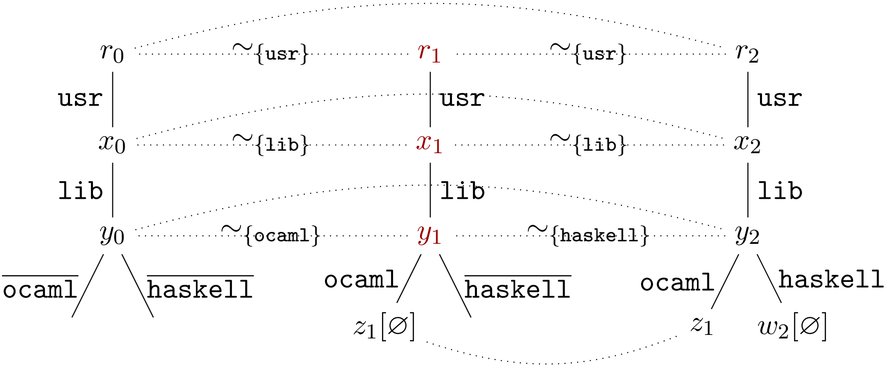

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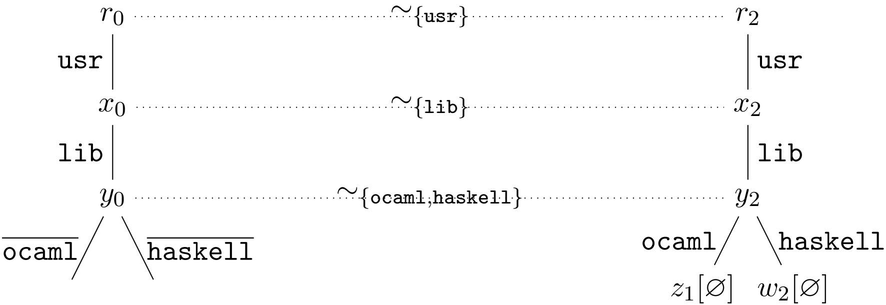

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First Order

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- In the example:

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- We can go for a weak quantifier elimination.

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- We can remove all quantifier blocks but one.
- If we know how to handle the last block, it's won.
  - in our case, we do for closed formula.

Conclusion

- CoLiS project: verifying Debian packages and their shell scripts.
- Feature trees with update to model modifications of filesystems.
- Incremental procedure to decide satisfiability of an existential fragment.
- Extends to first order via weak quantifier elimination.

- Article:

Nicolas Jeannerod, Ralf Treinen. *Deciding the First-Order Theory of an Algebra of Feature Trees with Updates*. IJCAR 2018

- Thank you for your attention! Any questions?