Formalising an intermediate language for POSIX shell

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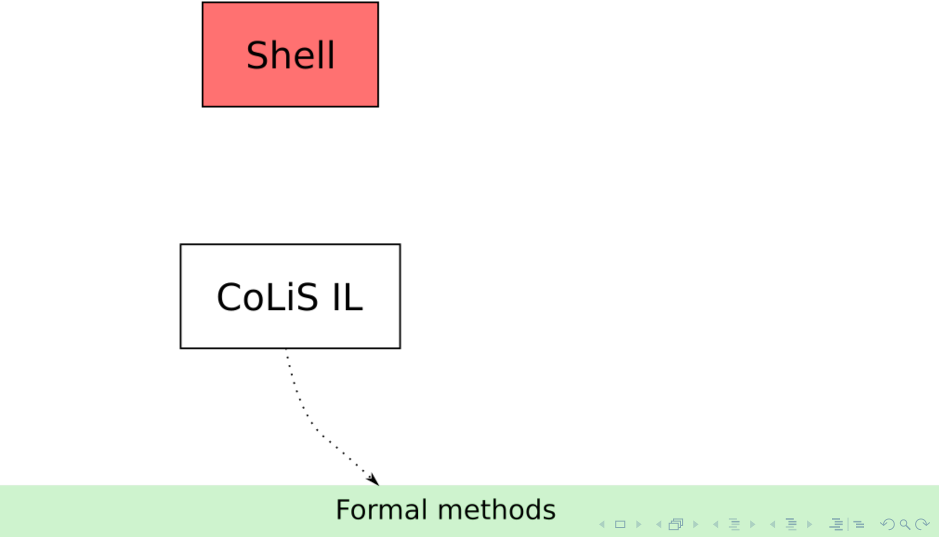
Big picture

Formal methods

Big picture



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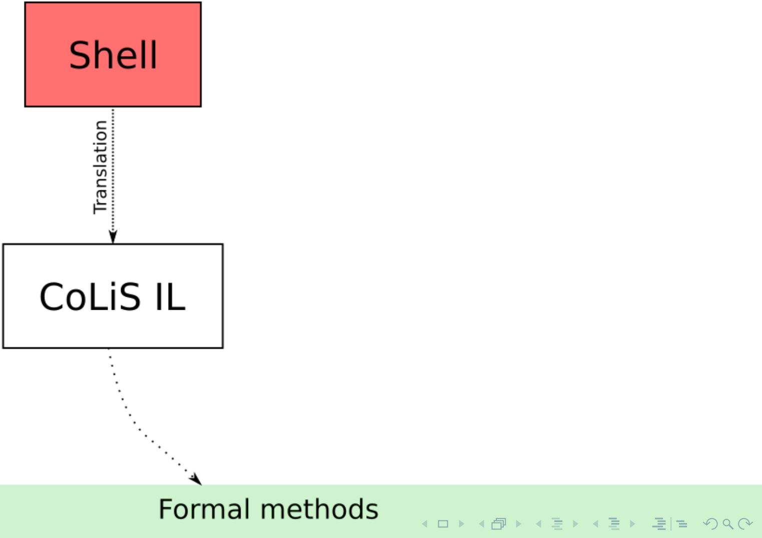

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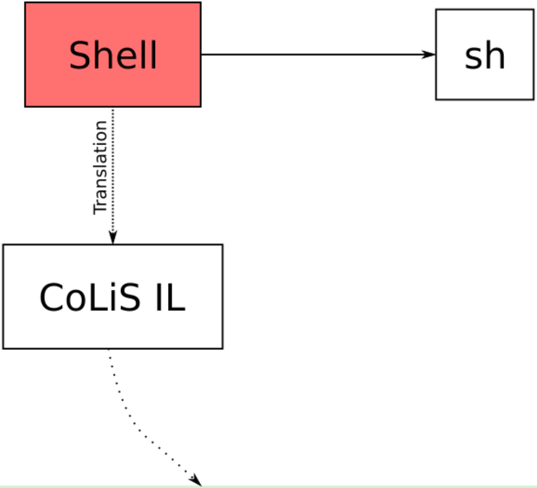
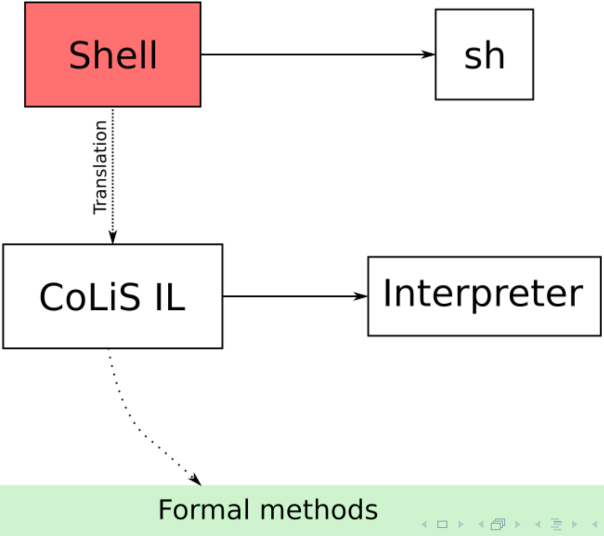


Formal methods

Big picture



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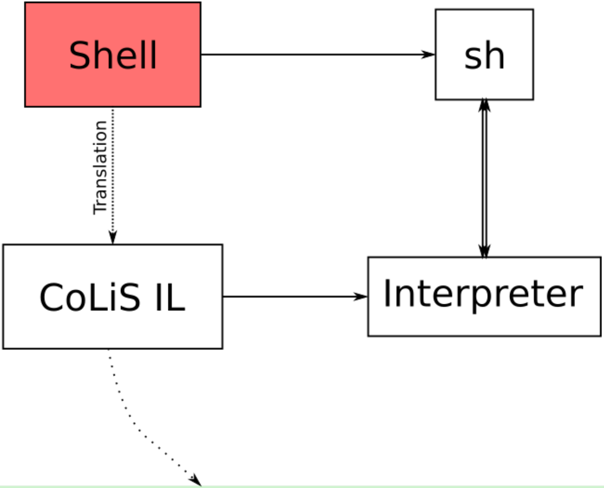

Formal methods

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Execute arbitrary strings

Execute commands from strings:

```
a='echo foo'
$a                 ## prints "foo"
```

or any code with `eval`:

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eval "if true; then echo foo; fi"
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Dynamic

Everything is dynamic:

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f () { g; }
g () { a=bar; }
a=foo
f
echo $a          ## prints "bar"
```

I tell ya, everything!

```
f () { echo $a; }
a=foo
a=bar f          ## prints "bar"
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All it can contain

- Literals
- Tildes
- Parameters (*i.e. variables*)
- Special parameters
- “Formatted” parameters
- Arithmetic
- Globs
- Commands
- Quotes

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```
~/Pictures        ~user/Pictures:~/Download
```

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$foo        $bar
```

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```
$@        $*        $1, $2, ...
```

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```
${foo:-bar}    ${foo-baz}
 ${foo%.*}     ${foo##*/}
```

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```
    $((1 + x + $x))
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```
/home/[!a]*    *.ml  *.ml?
```

- Commands
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```
$(echo foo)
`echo \`echo foo\``
$(which curl)
```

- Quotes

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```
foo='my file'
rm $foo '$foo' "$foo"
```

Dirty uses

Abused to represent both strings and lists of strings:

```
path='/home'
path="$path/nicolas"     ## "/home/nicolas"
args='-l -a'
args="$args -h"          ## ["-l"; "-a"; "-h"]
ls $args $path
```

Or lists separated by something else than space:

```
PATH='/usr/local/bin:/usr/bin:/bin'
IFS=:
for dir in $PATH; do
    echo $dir
done
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Dynamic changes in the semantics: IFS

```
file='git-sucks'
rm -r $file     ## deletes "git-sucks"
IFS=-
rm -r $file     ## deletes "git" and "sucks"
```

Here is what happens:

1. The parsing gives us `["rm"; "-r"; "$file"]`;
2. We apply *parameter expansion* and get `["rm"; "-r"; "git-sucks"]`;
3. We apply *field splitting*, **but only** where we just applied the parameter expansion: `[["rm"]; ["-r"]; ["git"; "sucks"]]`;
4. We flatten everything: `["rm"; "-r"; "git"; "sucks"]`;
5. We evaluate that so-called simple command.

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Dynamic changes in the semantics: set

With `set`:

- -a Every assignment becomes an `export`;
- -C > no longer overwrite existing files. >| still does;
- -e The shell shall exit immediately when a command fails, when this failure is not caught;
- -f Disables pathname expansion;
- -u The shell shall fail when expanding parameters that are unset.

It makes you wonder why most of these options are *disabled* by default.

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```
echo foo > file
set -C
echo bar > file      ## fails
echo baz >| file     ## succeeds
```

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set -e
! true ; echo foo     ## prints "foo"
false  ; echo foo     ## exists
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echo *     ## prints the files in $PWD
set -f
echo *     ## prints "*"
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set -u
rm -rf "$dir"/     ## fails
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Behaviours

Let us play with `exit`:

```
exit | echo 'foo'         ## prints "foo"
exit || echo 'foo'        ## exits
exit & echo 'foo'         ## prints "foo"
exit && echo 'foo'        ## exits

echo 'foo' | exit         ## does nothing
echo 'foo' || exit        ## prints "foo"
echo 'foo' & exit         ## prints "foo" and exits
echo 'foo' && exit        ## prints "foo" and exits
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The incredible story of set -e

> *When this option is on, when any command fails, the shell immediately shall exit, as if by executing the exit special built-in utility with no arguments, with the following exceptions: [...]*

Snippet 1:

```
false; echo 'foo'
```

Snippet 2:

```
{ false; echo 'foo'; } && echo 'bar'
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Snippet 3:

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- Intermediate language for a subset of shell;
- Not a replacement of shell;
- Well-defined and easily understandable semantics:
  - Some typing (strings vs. string lists),
  - Variables and functions declared in a header,
  - Dangers made more explicit;
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  - We must be convinced that it shares the same semantics as the shell,
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Syntax

Programs	p	$::=$	$\mathit{vdecl}^*\ \mathit{pdecl}^*$ **program** t
Variables decl.	vdecl	$::=$	**varstring** x_s $\mid$ **varlist** x_l
Procedures decl.	pdecl	$::=$	**proc** c **is** t
Terms	t	$::=$	**true** $\mid$ **false** $\mid$ **fatal**
		$\mid$	**return** t $\mid$ **exit** t
		$\mid$	$x_s := s$ $\mid$ $x_l := l$
		$\mid$	$t\ ;\ t$ $\mid$ **if** t **then** t **else** t
		$\mid$	**for** x_s **in** l **do** t $\mid$ **while** t **do** t
		$\mid$	**process** t $\mid$ **pipe** t **into** t
		$\mid$	**call** l $\mid$ **shift**

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\\
\text{Terms} & t & ::= & \textbf{true} \ \mid \ \textbf{false} \ \mid \ \textbf{fatal} \\
 & & \mid & \textbf{return } t \ \mid \ \textbf{exit } t \\
 & & \mid & x_s := s \ \mid \ x_l := l \\
 & & \mid & t \ ; \ t \ \mid \ \textbf{if } t \textbf{ then } t \textbf{ else } t \\
 & & \mid & \textbf{for } x_s \textbf{ in } l \textbf{ do } t \ \mid \ \textbf{while } t \textbf{ do } t \\
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\text{Terms} & t & ::= & \textbf{true} \mid \textbf{false} \mid \textbf{fatal} \\
& & \mid & \textbf{return}\ t \mid \textbf{exit}\ t \\
& & \mid & x_s := s \mid x_l := l \\
& & \mid & t\ ;\ t \mid \textbf{if}\ t\ \textbf{then}\ t\ \textbf{else}\ t \\
& & \mid & \textbf{for}\ x_s\ \textbf{in}\ l\ \textbf{do}\ t \mid \textbf{while}\ t\ \textbf{do}\ t \\
& & \mid & \textbf{process}\ t \mid \textbf{pipe}\ t\ \textbf{into}\ t \\
& & \mid & \textbf{call}\ l \mid \textbf{shift}
\end{array}
$$

Syntax

$$
\begin{array}{lrcl}
\text{Programs} & p & ::= & \mathit{vdecl}^* \ \mathit{pdecl}^* \ \textbf{program} \ t \\
\text{Variables decl.} & \mathit{vdecl} & ::= & \textbf{varstring} \ x_s \ \mid \ \textbf{varlist} \ x_l \\
\text{Procedures decl.} & \mathit{pdecl} & ::= & \textbf{proc} \ c \ \textbf{is} \ t \\
 & & & \\
\text{Terms} & t & ::= & \textbf{true} \ \mid \ \textbf{false} \ \mid \ \textbf{fatal} \\
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 & & \mid & \textbf{for} \ x_s \ \textbf{in} \ l \ \textbf{do} \ t \ \mid \ \textbf{while} \ t \ \textbf{do} \ t \\
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$$
\begin{array}{llll}
\text{String expressions} & s & ::= & \textbf{nil}_s \mid f_s :: s \\
\text{String fragments} & f_s & ::= & \sigma \mid x_s \mid n \mid t \\
\text{List expressions} & l & ::= & \textbf{nil}_l \mid f_l :: l \\
\text{List fragments} & f_l & ::= & s \mid \textbf{split}\ s \mid x_l
\end{array}
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Semantic judgements

$$t_{/\Gamma} \Downarrow \sigma \star b_{/\Gamma'}$$

A *context* Γ contains:

- flags?
- a file system,
- the standard input,
- the arguments line,
- environments for string and list variables,
- an environment for procedures.

A *behaviour* b can be
True, False, Fatal, Return (True|False) or Exit (True|False).

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Semantic rules – Branching

Branching-True

$$\frac{t_{1/\Gamma} \Downarrow \sigma_1 \star b_{1/\Gamma_1} \qquad b_1 = \mathsf{True} \qquad t_{2/\Gamma_1} \Downarrow \sigma_2 \star b_{2/\Gamma_2}}{(\textbf{if } t_1 \textbf{ then } t_2 \textbf{ else } t_3)_{/\Gamma} \Downarrow \sigma_1\sigma_2 \star b_{2/\Gamma_2}}$$

Branching-False

$$\frac{t_{1/\Gamma} \Downarrow \sigma_1 \star b_{1/\Gamma_1} \qquad b_1 \in \{\mathsf{False}, \mathsf{Fatal}\} \qquad t_{3/\Gamma_1} \Downarrow \sigma_3 \star b_{3/\Gamma_3}}{(\textbf{if } t_1 \textbf{ then } t_2 \textbf{ else } t_3)_{/\Gamma} \Downarrow \sigma_1\sigma_3 \star b_{3/\Gamma_3}}$$

Branching-Exception

$$\frac{t_{1/\Gamma} \Downarrow \sigma_1 \star b_{1/\Gamma_1} \qquad b_1 \in \{\mathsf{Return}\ \_, \mathsf{Exit}\ \_\}}{(\textbf{if } t_1 \textbf{ then } t_2 \textbf{ else } t_3)_{/\Gamma} \Downarrow \sigma_1 \star b_{1/\Gamma_1}}$$

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Table of Contents

Formalisation

- Formalised in the proof environment Why3:
  - The syntax becomes an algebraic data type,
  - The semantics become an inductive predicate;
- Interpreter proven sound and complete:
  - Written in a “natural way”,
  - Helps detecting potential mistakes in the semantics,
  - More easily readable than the semantics,
  - Allows us to validate the translation by testing.

Formalisation

- Formalised in the proof environment Why3:
  - The syntax becomes an algebraic data type,

```
type term = TTrue | TFalse | TFatal
  | TReturn term | TExit term
  | TSeq term term | TIf term term term
  | ...
```

  - The semantics become an inductive predicate;

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```
inductive eval_term term context
                        string behaviour context =
| EvalT_Seq_Normal : forall t₁ Γ σ₁ b₁ Γ₁ t₂ σ₂ b₂ Γ₂.
    eval_term t₁ Γ σ₁ (BNormal b₁) Γ₁ ->
    eval_term t₂ Γ₁ σ₂ b₂ Γ₂ ->
    eval_term (TSeq t₁ t₂) Γ (concat σ₁ σ₂) b₂ Γ₂
```

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exception EFatal context
exception EReturn (bool, context)
exception EExit (bool, context)

let rec interp_term (t: term) (Γ: context)
              (stdout: ref string) : (bool, context)
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```
  | TIf t1 t2 t3 ->
    let (b1 , Γ1) =
      try interp_term t1 Γ stdout
      with EFatal Γ1 -> (false, Γ1) end
    in
    interp_term (if b1 then t2 else t3) Γ1 stdout
```

  - Allows us to validate the translation by testing.

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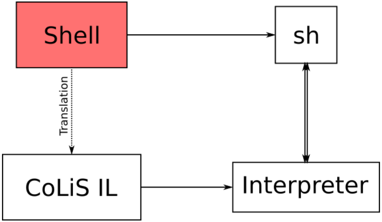

Soundness of the interpreter

We write $t_{/\Gamma} \mapsto \sigma \star b_{/\Gamma'}$ for: "*on the input consisting of* t, Γ *and a reference, the interpreter writes* σ *at the end of that reference and terminates:*

- *normally and outputs* (b, Γ')*;*
- *with an exception corresponding to the behaviour* b *that carries* Γ'*.*"

Theorem (Soundness of the interpreter)

For all t, Γ, σ, b *and* Γ'*: if*

$$t_{/\Gamma} \mapsto \sigma \star b_{/\Gamma'}$$

then

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Soundness of the interpreter in Why3

```
let rec interp_term (t: term) (Γ: context)
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  diverges

  returns { (b, Γ') -> exists σ.
    !stdout = concat (old !stdout) σ
    /\ eval_term t Γ σ (BNormal b) Γ' }

  raises  { EFatal Γ' -> exists σ.
    !stdout = concat (old !stdout) σ
    /\ eval_term t Γ σ BFatal Γ' }

  raises { EReturn (b, Γ') -> exists σ.
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Completeness of the interpreter in Why3

```
lemma functionality: forall t Γ σ1 σ2 b1 b2 Γ1 Γ2.
  eval_term t Γ σ1 b1 Γ1 ->
  eval_term t Γ σ2 b2 Γ2 ->
 σ1 = σ2 /\ b1 = b2 /\ Γ1 = Γ2
```

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let rec interp_term (t: term) (Γ: context)
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  requires { exists σ b Γ'. eval_term t Γ σ b Γ' }
```

```
  variant { ??? }
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Table of Contents

Why it is hard

- `stdout` is a reference.

```
exists σ. !stdout = concat (old !stdout) σ
       /\ eval_term t Γ σ (BNormal b) Γ'
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  - We cannot provide a witness as a return value here, because of exceptions,
  - We (c|sh)ould change it to something more structured.
  - We decided to use superposition provers.

- We need a variant:
  - The term?
  - The derivation tree of the hypothesis?
  - The height of the derivation tree?
  - The size of the derivation tree?
  - What then?

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- We need a variant:
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  - The size of the derivation tree?
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Why it is hard

- `stdout` is a reference.
- We need a variant:
  - The term? **No.**

$$\frac{\begin{array}{c} t_1{}_{/\Gamma} \Downarrow \sigma_1 \star b_1{}_{/\Gamma_1} \qquad b_1 = \text{True} \\ t_2{}_{/\Gamma} \Downarrow \sigma_2 \star b_2{}_{/\Gamma_2} \qquad b_2 \in \{\text{True}, \text{False}\} \\ (\textbf{while } t_1 \textbf{ do } t_2)_{/\Gamma_2} \Downarrow \sigma_3 \star b_3{}_{/\Gamma_3} \end{array}}{(\textbf{while } t_1 \textbf{ do } t_2)_{/\Gamma} \Downarrow \sigma_1\sigma_2\sigma_3 \star b_3{}_{/\Gamma_3}}$$

  - The derivation tree of the hypothesis?
  - The height of the derivation tree?
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Why it is hard

- `stdout` is a reference.
- We need a variant:
  - The term? **No.**
  - The derivation tree of the hypothesis?
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  - The size of the derivation tree?
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Why it is hard

- `stdout` is a reference.
- We need a variant:
  - The term? **No.**
  - The derivation tree of the hypothesis?
    - True, but we cannot manipulate them in Why3.
  - The height of the derivation tree?
  - The size of the derivation tree?
  - What then?

Why it is hard

- `stdout` is a reference.
- We need a variant:
  - The term? **No.**
  - The derivation tree of the hypothesis? **True, but no.**
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Skeletons

We add a `skeleton` type:

```
type skeleton =
  | S0
  | S1 skeleton
  | S2 skeleton skeleton
  | S3 skeleton skeleton skeleton
```

It represents the "shape" of the proof.

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Put them everywhere – In the predicate

```
inductive eval_term term context
                    string behaviour context skeleton =

  | EvalT_Seq_Normal : forall t₁ Γ σ₁ b₁ Γ₁ t₂ σ₂ b₂ Γ₂ sk1 sk2.
    eval_term t₁ Γ σ₁ (BNormal b₁) Γ₁ sk1 ->
    eval_term t₂ Γ₁ σ₂ b₂ Γ₂ sk2 ->
    eval_term (TSeq t₁ t₂) Γ (concat σ₁ σ₂) b₂ Γ₂ (S2 sk1 sk2)

  | EvalT_Seq_Error : forall t₁ Γ σ₁ b₁ Γ₁ t₂ sk.
    eval_term t₁ Γ σ₁ b₁ Γ₁ sk ->
    (match b₁ with BNormal _ -> false | _ -> true end) ->
    eval_term (TSeq t₁ t₂) Γ σ₁ b₁ Γ₁ (S1 sk)
```

Put them everywhere – In the contract

```
let rec interp_term (t: term) (Γ: context)
                    (stdout: ref string) (ghost sk: skeleton)
                    : (bool, context)

  requires { exists s b g'. eval_term t g s b g' sk }

  variant { sk }

  returns { (b, Γ') -> exists σ.
    !stdout = concat (old !stdout) σ
    /\ eval_term t Γ σ (BNormal b)  Γ' sk }
```

Define some helpers

```
let ghost skeleton12 (sk: skeleton)
  requires { match sk with S1 _ | S2 _ _ -> true | _ -> false e
  ensures { match sk with S1 sk1 | S2 sk1 _ -> result = sk1 | _
  = match sk with S1 sk1 | S2 sk1 _ -> sk1 | _ -> absurd end
```

The following:

```
  let ghost sk1 = skeleton12 sk in
```

reads: *"We know that* `sk` *can only have one or two premises and we name the first one* `sk1`*."*

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Put them everywhere – In the code

```
| TSeq t1 t2 ->
  let ghost sk1 = skeleton12 sk in
  let (_, Γ1) = interp_term t1 Γ stdout sk1 in
  let ghost (_, sk2) = skeleton2 sk in
  interp_term t2 Γ1 stdout sk2

| TIf t1 t2 t3 ->
  let (b1, Γ1) =
    try
      let ghost sk1 = skeleton12 sk in
      interp_term t1 Γ stdout sk1
    with
      EFatal Γ' -> (false, Γ')
    end
  in
  let ghost (_, sk2) = skeleton2 sk in
  interp_term (if b1 then t2 else t3) Γ1 stdout sk2
```

Put them everywhere – In the code

```
| TSeq t₁ t₂ ->
  let ghost sk1 = skeleton12 sk in
  let (_, Γ₁) = interp_term t₁ Γ stdout sk1 in
  let ghost (_, sk2) = skeleton2 sk in
  interp_term t₂ Γ₁ stdout sk2
```

```
| TIf t₁ t₂ t₃ ->
  let (b₁, Γ₁) =
    try
      let ghost sk1 = skeleton12 sk in
      interp_term t₁ Γ stdout sk1
    with
      EFatal Γ' -> (false, Γ')
    end
  in
  let ghost (_, sk2) = skeleton2 sk in
  interp_term (if b₁ then t₂ else t₃) Γ₁ stdout sk2
```

And it's all green!

	Soundness	Completeness
Proof obligations	117	233
Time (seconds)	190	510

Other things about skeletons

- Generalisable, if we want more than the shape;
- Help in writing recursion in case of mutually recursive types;
- Can really be added automatically to inductive predicates;
- Works because:
  - the order of the premises is the order of the execution,
  - the proof tree looks pretty much like the recursive calls tree.

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Thank you for your attention!

Questions? Comments? Suggestions?

Claude Marché, Nicolas Jeannerod and Ralf Treinen
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VSTTE, July 2017